The 'velocity of shrinkage' for points on the particle surface is easily obtained as

$$\frac{ds_u}{dt} = \frac{M}{\rho} N_{us} = \frac{M}{\rho} k_c C_\infty. \tag{12}$$

Where ds_u is the element of length perpendicular to the particle surface and this is constant over the surface of the particle. As a result, the ratio b/a between minor and major axes of the ellipse decreases in the process of reaction and so the burning particle deviates increasingly from a sphere as the reaction proceeds. Indeed, in any period of time, b_i and a_i are reduced by the same amount, say s, and for $b_i < a_i$ it is obvious that $(b_i/a_i) > (b_i - s)/(a_i - s)$.

The evolution of particle shape may be easily obtained from the original ellipsoid by 'peeling off' layers of constant thickness.

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Average Nusselt number on the downward-facing heated plate

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INTRODUCTION

IN A RECENT paper Schulenberg [1] determined analytically the stagnation point heat transfer coefficient for natural convection on the horizontal downward-facing heated plate. In his Fig. 9 he plotted 12 points based upon experimental data for the ratio of average to stagnation point Nusselt number. Hatfield and Edwards [2] had correlated average Nusselt number on the basis of a virtual extension, x_0, of the boundary layer, as shown in Fig. 1, to allow for the finite thickness of the boundary layer as it flowed around the corner of the plate, or outwards onto a horizontal adiabatic extension of length L_a. In this note we apply the virtual displacement concept to find a closed-form correlation based upon Schulenberg's analysis and compare it to the previous correlation [2].

ANALYTICAL BASIS

Given a local Nusselt number Nu_x that varies with local Ra_x, where x is measured from the virtual edge, the average Nu_L based upon total length L is

$$Nu_L = \frac{\bar{h}L}{k} = \frac{L}{k}\frac{2}{L}\int_{x_0}^{x_0+L/2}\left(\frac{k}{x}Nu_x\right)dx. \tag{1}$$

In keeping with the approximate boundary-layer theory of Singh *et al.* [3] and the stagnation point solution of Schulenberg [1], a one-fifth power relationship is assumed

$$Nu_x = C'(Pr)\, Ra_x^{1/5}. \tag{2}$$

The result of Schulenberg for the stagnation point can be expressed as a Nusselt number based upon total length L

$$Nu_0 = \frac{h_0 L}{k} = 2^{2/5} C(Pr)\, Ra_L^{1/5} \tag{3}$$

where for the infinite isothermal strip Schulenberg gives

$$C(Pr) = \frac{0.571 Pr^{1/5}}{(1+1.156 Pr^{3/5})^{1/3}}. \tag{4}$$

Coefficient $C'(Pr)$ in equation (2) is related to $C(Pr)$ in equation (3) by equating h_0 to the local heat transfer coefficient at $x_0 + L/2$. Equation (2) may then be substituted into equation (1) and the integration carried out. The result is

$$\frac{\bar{h}}{h_0} = \frac{Nu_L}{Nu_0} = \frac{5}{3}(1+\zeta)^{2/5}[(1+\zeta)^{3/5} - \zeta^{3/5}] \tag{5}$$

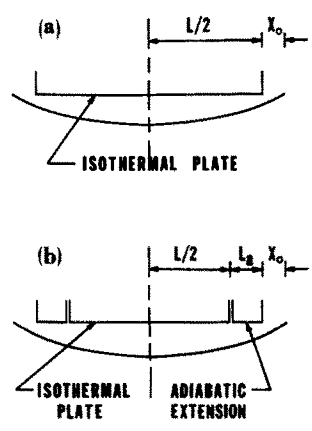

FIG. 1. Schematic of the boundary layer on a heated horizontal plate facing down. (a) Bare edges; (b) adiabatic extensions.

NOMENCLATURE

c	specific heat capacity
$C(Pr)$	dimensionless parameter defined by equation (4)
$C'(Pr)$	dimensionless parameter in equation (2)
C_1, C_2, C_3, C_4	empirical constants in ref. [2]
$\bar{h}$	average heat transfer coefficient defined by equation (1)
h	local heat transfer coefficient
h_0	heat transfer coefficient at the stagnation point
k	thermal conductivity
L	short side length of plate
L_a	length of adiabatic extension
Nu_L	average Nusselt number, $\bar{h}L/k$
Nu_x	local Nusselt number, hx/k
Nu_0	Nusselt number at the stagnation point based on L, h_0L/k
Pr	Prandtl number, ν/α
Ra_L	Rayleigh number, $g\beta TL^3\nu^{-1}\alpha^{-1}$
Ra_x	local Rayleigh number, $g\beta Tx^3\nu^{-1}\alpha^{-1}$
T	temperature
W	long side length of plate
x_0	edge displacement length (virtual extension).

Greek symbols

α	thermal diffusivity, $k/\rho c$
β	coefficient of thermal expansion, $-(1/\rho)\partial\rho/\partial T$
ζ	dimensionless parameter in equation (4), $2x_0/L$
ΔT	wall to free-stream temperature difference
ν	kinematic viscosity
ρ	fluid density.

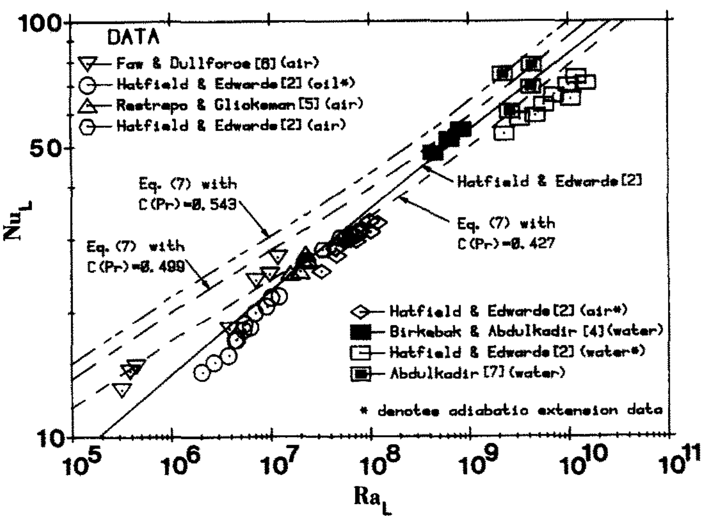

FIG. 2. Comparison of correlations with experimental data (the asterisk denotes data with adiabatic extensions plotted on a length basis of $L+2L_a$).

where $\zeta = 2x_0/L$. In the limit of small ζ the average to stagnation point ratio goes to (5/3) $(1-\zeta^{3/5})$. In the limit of large ζ it goes to $[1+(5\zeta)^{-1}]$.

CORRELATION

Equation (5) permits Schulenberg's $\bar{h}/h_0$ vs Ra figure to be recast as ζ vs Ra_L where L is again based upon the total length. Disregarding the low Prandtl number point, we choose to fit the data with

$$\zeta = 2.87\times10^{-5}\,Ra_L^{3/5}. \tag{6}$$

The $(1+0.38L/W)$ aspect ratio correction of ref. [2] is retained where $W \geqslant L$ is the long dimension of a rectangle. Accordingly the expression for Nu_L can be written as

$$Nu_L = 2^{2/5}C(Pr)Ra_L^{1/5}(1+0.38L/W)(\bar{h}/h_0) \tag{7}$$

where $\bar{h}/h_0$ is given by the RHS of equation (5) with ζ given by equation (6). The correlation, equation (7), is thus made with only the two arbitrary constants in equation (6).

Figure 2 shows data from refs. [2, 4–7] compared to equation (7). Data points are shown corrected to squares. Data with small adiabatic extensions are plotted as Nu based upon $L+2L_a$ vs Rayleigh number similarly based. Shown also is the correlation equation (12) of ref. [2] for $L_a/L = 0$. Note that the correlation constants in ref. [2] were scrambled; the correctly ordered values are $C_1 = 6.5$, $C_2 = 13.5$, $C_3 = 2.2$ and $C_4 = 0.38$. The values of $C(Pr) = 0.427, 0.499$ and 0.543 in Fig. 2 correspond to $Pr = 0.7$, 6, and 4800, respectively; thus air, water and oil.

The new correlation, equation (7) here, differs from the old correlation, equation (12) in ref. [2], in three respects. First, the old correlation has no Pr dependency except through Rayleigh number. Second, it indicates lower values of Nu_L at low values of Ra_L. Third, the old correlation made specific provision for adiabatic extensions with two additional empirical constants. The present correlation makes no specific provision, and the data with nonzero L_a plot somewhat low.

The data in the figure do not indicate unambiguously whether the strong Pr dependency in equation (7) really exists. Data for oil seem to have been reported only for 3 : 1

rectangles with adiabatically extended edges [2]. These data, plotted as *Nu* based upon $L+2L_a$ vs *Ra* similarly based, plot 45.6% lower than the new correlation, but only 23.9% lower than the old correlation, in the vicinity of $Ra_L = 5\times10^6$. The new correlation agrees well with Birkebak and Abdulkadir's data [4, 7] in the vicinity of $Ra_L = 10^9$ and $Ra_L = 4\times10^9$, respectively. The agreement is also good with the air data of Restrepo and Glicksman [5] and Faw and Dullforce [6] for $Pr = 0.7$, while the old correlation underpredicts the data of Faw and Dullforce at $Ra_L = 4\times10^5$.

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